J. Fluid Mech. (1991), *vol.* 231, *pp.* 689–693
Printed in Great Britain

REVIEWS

Méthodes des Éléments Finis pour les Fluides. By O. PIRONNEAU. Masson, 1988. 205 pp. 175F or £27.50.

Numerical Solution of Partial Differential Equations by the Finite Element Method. By C. JOHNSON. Cambridge University Press, 1987. 278 pp. £40 or $69.50 (hardback), £15 or $24.95 (paperback).

The series Recherches en Mathematiques Appliquées to which the book by Pironneau belongs consists of rapidly published, camera-ready monographs comprising edited research articles, conference proceedings and theses, aimed at a high-level audience. This book about finite-element methods presupposes a good background in numerical analysis, variational principles for partial differential equations, finite elements and basic fluid mechanics. It is founded on a series of lectures given recently to (the equivalent of M.Sc.) students at the Université de Pierre et Marie Curie, Paris, and is particularly suitable for graduate students and researchers wishing to specialize in both the algorithmic and theoretical aspects of computational fluid dynamics. So clear is the exposition that the text should present few problems to readers possessing only a basic command of the French language.

A broad spectrum of topics is covered, opening with a general résumé of the basic equations of fluid mechanics, and proceeding to chapters considering irrotational and weakly irrotational flows, convection–diffusion phenomena, the Stokes problem, the Navier–Stokes equations (incompressible and compressible), compressible Euler equations and the equations of Saint-Venant. A discussion of subsonic and transonic flows is included, and turbulence modelling is briefly discussed. Almost all of the problems discussed in these chapters are illustrated by results drawn from a very extensive and up-to-date reference list.

Attention is paid throughout to the important aspects of existence, uniqueness and regularity of solutions in the rigorous settings of abstract function spaces. The problem of variables 'at infinity' in unbounded domains is addressed. Specialist features, pertaining to explicit and implicit time-dependent formulations and to nonlinear phenomena, are described.

Since it is assumed that the reader is already familiar with the finite-element method, computational implementation details are frequently minimal or completely absent. The exception to this is the discussion of elemental degrees of freedom in the primitive-variable formulation of the Stokes problem. In all cases, however, the variational approach underlying each particular application is clearly discussed and, where necessary, a *pseudocode* algorithmic form is presented. The absence of programming details is not unconnected, one suspects, with the description in the appendix of the author's *MacFEM*™, a user-friendly, menu-driven (Pascal) finite-element software package dedicated to the Apple Macintosh™. *MacFEM*™ has applications in many of the areas covered in the earlier chapters; results for potential flows and shock phenomena are presented.

In summary, this is a well-balanced, readable, thorough text illustrating state-of-the-art finite-element approaches across a very broad range of applications in both theoretical and practical fluid mechanics. Indeed, the appendix contains excellent colour plates of advanced graphical results drawn from the most recent industrial applications. The combined presence, in one volume, of fundamental mathematical principles and extensive references illustrating current approaches makes this book a valuable contribution, at a research level, to the finite-element literature of computational fluid dynamics.

An alternative approach is adopted in Johnson's text, which is more introductory in nature, and is more suited to undergraduate or beginning graduate courses on computational methods for partial differential equations for students of pure and applied science. Whilst the emphasis in this work is to present the mathematical and numerical properties of the finite-element method as a general tool for the solution of partial differential equations, applications in many areas of both steady-state and time-dependent fluid and solid mechanics are presented.

Separate chapters are dedicated to the application of the finite-element method to elliptic, parabolic and hyperbolic problems with, as in much other literature in the field, a heavier theoretical bias towards the first of these. The concept of abstract function spaces, necessary for considerations of regularity and convergence of numerical solutions, is introduced where appropriate at just the right level so as not to detract from the main direction of the book. It is refreshing to see such information included in this way, so that all relevant material is to be found in one self-contained text.

Rigorous error analyses, and detailed suggestions for overcoming specific difficulties of computational implementations are clearly presented, with many arguments clarified by geometrical interpretations. It was pleasing to see a section containing on-hand information devoted to various direct and iterative algorithms for the solution of banded systems of linear equations. There is a brief section directing the reader to currently available general-purpose finite-element software packages, and reference is made to very recent developments in the area of *adaptive meshing* for steady-state elliptic problems.

The discussion of parabolic problems is pitched at a slightly higher level, not least because of the difficulties associated with the control of the inherent time-stepping process. Error estimates, leading to automatic time- *and* space-step control, are introduced, but for further information the reader is directed to more advanced references.

The streamline diffusion (shock capturing) method for the accurate resolution of first- and second-order hyperbolic (convection–diffusion) problems is discussed at some length, and comparisons are made with the classical approach of artificial viscosity. A discontinuous Galerkin method for the solution of a reduced (non-diffusive) problem is presented, considerable detail being paid to implementation.

There are brief sections on boundary-element (integral equation) methods, mixed finite-element methods, curved elements and numerical integration, finishing with a pot-pourri of applications to nonlinear problems, including both incompressible Euler and Navier–Stokes equations, and the compressible Burgers' equation.

This book has a 'fresh' feel to it, in so far as the author has managed to bring several different (but relevant) areas – functional analysis, numerical analysis, computing, variational calculus, fluid and solid mechanics – into a homogeneous style which is both exceptionally clear and stimulating to read. The section on automatic time-step control for parabolic problems was, at the time of going to press, state-of-the-art research. I would by no means limit my recommendation of this text to students; it is an excellent, self-contained, detailed, well-referenced volume which addresses a wide range of theoretical and practical aspects, and would therefore be valuable to researchers in many fields.

M. A. Kelmanson

SHORTER NOTICES

Les Modèles Asymptotiques de la Mécanique des Fluides I, II. By R. KH. ZEYTOUNIAN. Springer. Vol. I, 1986. 260 pp. DM 38; Vol. II, 1987. 315 pp. DM 53.

The contents of the chapters of this two-volume set of notes on modelling in fluid mechanics are as follows.

I. The exact model of Navier and Stokes (basic formulation, initial and boundary conditions, dimensionless parameters). II. 'Modèlisation' in the mechanics of Newtonian fluids (asymptotic for large and small Reynolds number, ordered with respect to large or small Mach number; global models, such as hydrodynamics, hypersonics, slow flow; asymptotic methods of matched expansions and multiple scales in simple examples). III. Formal considerations for singular perturbation methods (matched expansions and multiple scales, ordinary and partial differential equations, hyperbolic systems, ray theory). IV. The model of Navier (viscous flow, basically incompressible, but with weak compressible fields induced by specified temperature variations; the generalized Rayleigh problem, with compressibility corrections and matching to wave fields; general aspects of inner rotational field matching to outer wave fields; vorticity dynamics for viscous incompressible fluid; perturbations, stability theory, short-wave asymptotics). V. The Euler model (the Bernoulli and Helmholtz equations, invariants, hyperbolic systems, vortical flows, the Prandtl–Batchelor model, triple-decks, vortex sheets, high-aspect-ratio wings; shock waves, transonic similarity, unsteady gasdynamics, 'hyposonics' – incompressible rotational core plus outer wave field; flow through turbo-machines, actuator-disk theory as a singular perturbation problem). VI. Prandtl's model, boundary layers, regular coupling (Blasius solution, general boundary equations). VII. Stokes and Oseen flows (Stokes and Whitehead paradoxes, Oseen modification, unsteady flows). VIII. Rayleigh model (large Reynolds number and large Strouhal numbers, generalizations of the Rayleigh problem). IX. Triple-deck theory (basic triple-deck structure, applications to shock–boundary layer interaction, trailing-edge flow, flow over inhomogeneities, bluff-body separation, unsteady triple decks), X. Zeytounian model (small viscosity, small compressibility, Blasius problem, diffusion coefficients dependent on temperature). XI. Riley–Stuart model (acoustic streaming, various distinguished parameter scalings).

The text is in French, but is easily understandable by anyone with elementary French. Equations are handwritten, but very clearly done. Some readers may feel that there is inadequate emphasis placed on the physical interpretation of the results obtained. Nevertheless, in many of the areas covered in these two volumes there is a conspicuous lack of suitable expository material available elsewhere in the literature, and Professor Zeytounian's notes are to be welcomed for filling these gaps until fuller and more specialized accounts appear in book form.

Dynamics of Fluids in Hierarchical Porous Media. Edited by J. H. CUSHMAN. Academic, 1990. 505 pp. £48.

Theoreticians often assume that natural porous media such as soil and rock are statistically homogeneous and that the coefficient in the Darcy linear relation between pressure gradient and flux of fluid volume is spatially uniform. That assumption owes more to expediency than to observation, and there has been much research in recent years on the effects of the spatial heterogeneity. Different types of heterogeneity are associated with different lengthscales, and the 'hierarchy' referred to in the title extends all the way up to the scale of the Earth (as in atmospheric dynamics). It is not surprising that standard theories of transport in homogeneous porous media are often found to be quite inadequate when tested on natural formations.

This book is an outgrowth of lectures presented at a symposium on hierarchical porous media held in San Francisco late in 1988. The texts of the lectures have been reviewed, harmonized in general style and appearance, and reproduced as separate chapters of the book. It is not a text for the uninitiated, but those involved in research in this difficult field are likely to find some of the 17 chapters useful.

The Mathematical Theory of Non-uniform Gases. By S. Chapman and T. G. Cowling. Cambridge University Press, 1990. 423 pp. £19.50 or $32.50.

The third edition, published in 1970, of this classic book on the kinetic theory of gases is here reissued in paperback form. It includes a new and interesting foreword in which Professor C. Cercignani describes the history of the development of kinetic theory and the contributions by Daniel Bernoulli, Clausius, Maxwell, Hilbert, Chapman, Enskog, Burnett and Grad. He also shows how the book is still useful, as well as being historically important. There will not be many who wish to master the formidable methods needed for the accurate calculation of transport processes in gases, but the basic ideas are an essential part of fluid mechanics.

Theory of Macroscopic Systems. By C. Ouwerkerk. Springer, 1991. 245 pp. DM 48.

It takes a little time to discover what this book is really about. There is no 'theory' in the accepted sense, and the sub-title 'A unified approach for engineers, chemists and physicists' does not convey much. 'Macroscopic systems' appears to mean matter in bulk, treated as continua. Digging further in reveals that the purpose of the book is to present non-equilibrium thermodynamic and mechanical relations for a wide range of circumstances from a common set of 'balance equations' and in a common notation. Phenomenological molecular transport relations are prominent. Chemical reaction engineering is the field to which the relations presented appear to have most relevance. No actual physical problems are described. Fluid flow makes only a minor appearance, near the end of the book. There is no comment on the absence of a pressure gradient from the 'simplified transport equation' for momentum on page 229.

BASIC Fluid Mechanics. By J. J. Sharp. Butterworths, 1988. 139 pp. £9.95.
BASIC Hydrodynamics. By A. C. Thomson. Butterworths, 1987. 179 pp. £9.95.
BASIC Heat Transfer. By D. H. Bacon. Butterworths, 1989. 172 pp. £12.95.

These books come from a series with a common format. Each book starts with a chapter giving a summary of the BASIC programming language. The first-named uses GW BASIC, the others use BASIC for an Apple computer. Further chapters contain brief descriptions of subject matter, in some cases hardly more than 'revision notes', and in others just enough for an introduction. Each chapter concludes with BASIC programs. Their structure is described, notes are provided explaining details and, in most cases, sample output is provided. Certain programs presume a sophisticated BASIC which can find the inverse of a matrix. The chapters conclude with a set of problems involving use of the programs. The scope of the programs ranges from simple evaluation of a formula to solution of Laplace's equation and of the diffusion equation. In these latter cases there is a brief account of the numerical analysis involved.

The book on Fluid Mechanics is the most elementary of this trio and includes topics that appear in an introductory course for engineering students, e.g. hydrostatics, kinematics, dimensional analysis and simple force and momentum analyses. That on Hydrodynamics is primarily on potential flow, with only a brief section on boundary layers. It includes chapters on flow in a porous medium and free-surface flows. The topics in the book on Heat Transfer are conduction, convection, radiation, finned surfaces and heat exchangers.

CORRIGENDUM

Turbulent open-channel flows with variable depth across the channel

By Koji Shiono and Donald W. Knight

Journal of Fluid Mechanics, vol. 222 (1991), pp. 617–646

The expressions for β and η in equation (11) on p. 621 should read

$$\beta = \frac{\Gamma}{\rho g S_0 H},$$

$$\eta = -\frac{\Gamma}{\dfrac{(1+s^2)^{\frac{1}{2}}}{s}\rho\dfrac{f}{8}}.$$